ADITYA COLLEGE OF ENGINEERING AND TECHNOLOGY
Kamakshipura, Hesaraghatta Hobli, Yelahanka, Bengaluru, Karnataka, 560089
Affiliated to VTU, Belagavi, Approved by AICTE, New Delhi

**Department of Mathematics**
**QUESTION BANK**

| | | | |
|---|---|---|---|
| **Course** | Probability, Distributions and Statistics | **Course Code** | 1BMATCS301 |
| **Semester** | III | **Scheme** | 2025 |
| **Modules** | Module1: Modular Arithmetic & Module 2: Statistics | **Credits** | 4 |

| Q. No. | Question | Level | CO |
|---|---|---|---|
| **Module 1: Modular Arithmetic (CO1)** | | | |
| 1 | Find the least positive value of x such that (i) $71 \equiv x \pmod 8$, (ii) $78 + x \equiv 3 \pmod 5$, (iii) $89 \equiv (x + 3) \pmod 4$. | L2 | CO1 |
| 2 | Using congruences, find the remainder when (i) $349 \times 74 \times 36$ is divided by 3, and (ii) $175 \times 113 \times 53$ is divided by 11. | L2 | CO1 |
| 3 | Find the last digit of (i) $7^{2013}$, (ii) $13^{37}$, (iii) $7^{118}$. | L2 | CO1 |
| 5 | State the condition under which the linear congruence $ax \equiv b \pmod m$ has a solution. Hence solve (i) $11x \equiv 4 \pmod{25}$, (ii) $12x \equiv 6 \pmod{21}$, and (iii) $6x \equiv 9 \pmod{15}$, stating the number of incongruent solutions in each case. | L2 | CO1 |
| 6 | Solve the polynomial congruences (i) $x^2 + x + 1 \equiv 0 \pmod 7$, and (ii) $x^3 + x + 1 \equiv 0 \pmod{27}$ by first solving modulo 3 and then lifting the solution to modulo 9 and modulo 27. | L3 | CO1 |
| 7 | State the condition for the linear Diophantine equation $ax + by = c$ to have integer solutions. Solve (i) $13x + 17y = 5$ and write its general solution, and (ii) $70x + 112y = 168$. | L2 | CO1 |
| 8 | Determine whether the equation $21x + 14y = 147$ has solutions and find all its positive integer solutions. Also, in how many ways can ₹100 be paid using only ₹5 and ₹7 coins? | L3 | CO1 |
| 9 | Solve the systems of linear congruences (i) $2x + 6y \equiv 1 \pmod 7$, $4x + 3y \equiv 2 \pmod 7$, and (ii) $7x + 3y \equiv 10 \pmod{16}$, $2x + 5y \equiv 9 \pmod{16}$. | L3 | CO1 |
| 10 | State the Chinese Remainder (Remainder) theorem and use it to solve $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 2 \pmod 7$. | L2 | CO1 |
| 11 | Using the remainder theorem, solve the system $x \equiv 3 \pmod 5$, $x \equiv 2 \pmod 6$, $x \equiv 4 \pmod 7$. | L3 | CO1 |
| 12 | State and prove Fermat's little theorem. Hence (i) find the remainder when $7^{121}$ is divided by 13, (ii) find the remainder when $2^{1000}$ is divided by 13, (iii) show that $8^{30} - 1$ is divisible by 31, and (iv) find the remainder when $2^{23}$ is divided by 47. | L3 | CO1 |

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|---|---|---|---|
| 13 | Define Euler's totient function φ(n) and find φ(100) and φ(360). State Euler's theorem and use it to (i) find the remainder when $3^{100}$ is divided by 35, and (ii) find the last two digits of $7^{100}$. | L3 | CO1 |
| 14 | State and prove Wilson's theorem. Hence find the remainder when (i) 14! and (ii) 15! is divided by 17. | L3 | CO1 |
| 15 | Explain the RSA algorithm (key generation, encryption and decryption). (i) Encrypt the message STOP using RSA with key (2537, 13) using the primes 43 and 59 (take A = 00, B = 01, …, Z = 25 and use blocks of four digits). (ii) For p = 5, q = 11 and e = 7, find the private key d. | L3 | CO1 |

| Q. No. | Question | Level | CO |
|---|---|---|---|
| | **Module 2: Statistics (CO2)** | | |
| 16 | (a) Fit a straight line y = ax + b by the method of least squares to the following data and hence estimate y at x = 30.<br>**x**: 5, 10, 15, 20, 25<br>**y**: 16, 19, 23, 26, 30<br>(b) Fit a straight line to the following data.<br>**x**: 1, 3, 4, 6, 8, 9, 11, 14<br>**y**: 1, 2, 4, 4, 5, 7, 8, 9 | L3 | CO2 |
| 17 | (a) Fit a second degree parabola $y = a + bx + cx^2$ in the least squares sense to the following data and estimate y at x = 6.<br>**x**: 1, 2, 3, 4, 5<br>**y**: 10, 12, 13, 16, 19<br>(b) Fit a parabola to the following data.<br>**x**: 0, 1, 2, 3, 4<br>**y**: 1, 5, 10, 22, 38 | L3 | CO2 |
| 18 | Fit a parabola of second degree $y = ax^2 + bx + c$ to the following data.<br>**x**: 0, 1, 2, 3, 4<br>**y**: 1, 1.8, 1.3, 2.5, 2.3 | L3 | CO2 |
| 19 | Reduce the curve $y = ax^b$ to linear form and write its normal equations. Hence fit the curve to the following data and estimate y at x = 7.<br>**x**: 1, 2, 3, 4, 5, 6<br>**y**: 2.98, 4.26, 5.21, 6.1, 6.8, 7.5 | L3 | CO2 |

<table>
<tr><th>Q. No.</th><th>Question</th><th>Level</th><th>CO</th></tr>
<tr><td>20</td><td>Reduce the curve $v = at^b$ to linear form and write its normal equations. Hence fit the curve by least square method.
<table>
<tr><td>v</td><td>350</td><td>400</td><td>500</td><td>600</td></tr>
<tr><td>t</td><td>61</td><td>26</td><td>7</td><td>26</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
<tr><td>21</td><td>Compute the coefficient of correlation and the equations of the lines of regression for the following data. Also find the acute angle between the two lines of regression.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>2</td><td>5</td><td>3</td><td>8</td><td>7</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
<tr><td>22</td><td>Define Karl Pearson’s coefficient of correlation and state its properties. (a) Find the coefficient of correlation for the two variables x and y.
<table>
<tr><td>x</td><td>92</td><td>89</td><td>87</td><td>86</td><td>83</td><td>77</td><td>71</td><td>63</td><td>53</td><td>50</td></tr>
<tr><td>y</td><td>86</td><td>83</td><td>91</td><td>77</td><td>68</td><td>85</td><td>52</td><td>82</td><td>37</td><td>57</td></tr>
</table>
(b) Also find r for the following data.
<table>
<tr><td>x</td><td>10</td><td>12</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>10</td><td>14</td><td>15</td></tr>
<tr><td>y</td><td>17</td><td>16</td><td>15</td><td>12</td><td>10</td><td>9</td><td>8</td><td>15</td><td>13</td><td>12</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
<tr><td>23</td><td>Obtain the lines of regression and hence find the coefficient of correlation for the following data.
<table>
<tr><td>x</td><td>1</td><td>3</td><td>4</td><td>2</td><td>5</td><td>8</td><td>9</td><td>10</td><td>13</td><td>15</td></tr>
<tr><td>y</td><td>8</td><td>6</td><td>10</td><td>8</td><td>12</td><td>16</td><td>16</td><td>10</td><td>32</td><td>32</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
<tr><td>24</td><td>The lines of regression of a bivariate distribution are $2x + 3y + 1 = 0$ and $x + 6y - 4 = 0$. Identify which line is the regression of y on x and which is of x on y, and compute $\bar{x}$, $\bar{y}$ and r.</td><td>L3</td><td>CO2</td></tr>
<tr><td>25</td><td>The two regression lines are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Find $\bar{x}$, $\bar{y}$ and r. Also find $\sigma_y$ if $\sigma_x = 3$.</td><td>L3</td><td>CO2</td></tr>
<tr><td>26</td><td>If $\theta$ is the angle between the two lines of regression, show that $\tan\theta = ((1 - r^2)/r) \cdot (\sigma_x\sigma_y / (\sigma_x^2 + \sigma_y^2))$. Explain the cases $r = 0$ and $r = \pm 1$.</td><td>L2</td><td>CO2</td></tr>
<tr><td>27</td><td>Define Spearman’s rank correlation coefficient. Ten students obtained the following percentage of marks in two subjects x and y. Compute the rank correlation coefficient.
<table>
<tr><td>x</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr>
<tr><td>y</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
</table>

<table>
<tr><th>Q. No.</th><th>Question</th><th>Level</th><th>CO</th></tr>
<tr><td>28</td><td>Ten competitors in a beauty contest are ranked by two judges A and B as follows. Calculate the rank correlation coefficient.
<table>
<tr><td>Competitor</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Judge A</td><td>1</td><td>6</td><td>5</td><td>10</td><td>3</td><td>2</td><td>4</td><td>9</td><td>7</td><td>8</td></tr>
<tr><td>Judge B</td><td>6</td><td>4</td><td>9</td><td>8</td><td>1</td><td>2</td><td>3</td><td>10</td><td>5</td><td>7</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
<tr><td>29</td><td>(a) Compute the rank correlation coefficient (with the correction for tied ranks) for the following data.
<table>
<tr><td>x</td><td>68</td><td>64</td><td>75</td><td>50</td><td>64</td><td>80</td><td>75</td><td>40</td><td>55</td><td>64</td></tr>
<tr><td>y</td><td>62</td><td>58</td><td>68</td><td>45</td><td>81</td><td>60</td><td>68</td><td>48</td><td>50</td><td>70</td></tr>
</table>
(b) Also calculate it for the following data (ties in both series).
<table>
<tr><td>x</td><td>6</td><td>4</td><td>3</td><td>1</td><td>2</td><td>7</td><td>9</td><td>8</td><td>10</td><td>5</td></tr>
<tr><td>y</td><td>4</td><td>1</td><td>6</td><td>7</td><td>5</td><td>8</td><td>10</td><td>9</td><td>3</td><td>2</td></tr>
</table></td><td>L3</td><td>CO2</td></tr>
</table>